General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MPC1

Unit Pure Core 1

Monday 14 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The point A has coordinates $(-3, 2)$ and the point B has coordinates $(7, k)$.

The line AB has equation $3x + 5y = 1$.

(a) (i) Show that $k = -4$. *(1 mark)*

(ii) Hence find the coordinates of the midpoint of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) A line which passes through the point A is perpendicular to the line AB. Find an equation of this line, giving your answer in the form $px + qy + r = 0$, where p, q and r are integers. *(3 marks)*

(d) The line AB, with equation $3x + 5y = 1$, intersects the line $5x + 8y = 4$ at the point C. Find the coordinates of C. *(3 marks)*

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2 A bird flies from a tree. At time t seconds, the bird's height, y metres, above the horizontal ground is given by

$$y = \frac{1}{8}t^4 - t^2 + 5, \quad 0 \leqslant t \leqslant 4$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}t}$. *(2 marks)*

(b) (i) Find the rate of change of height of the bird in metres per second when $t = 1$. *(2 marks)*

(ii) Determine, with a reason, whether the bird's height above the horizontal ground is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the value of $\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$ when $t = 2$. *(2 marks)*

(ii) Given that y has a stationary value when $t = 2$, state whether this is a maximum value or a minimum value. *(1 mark)*

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3 (a) (i) Express $\sqrt{18}$ in the form $k\sqrt{2}$, where k is an integer. *(1 mark)*

(ii) Simplify $\dfrac{\sqrt{8}}{\sqrt{18} + \sqrt{32}}$. *(3 marks)*

(b) Express $\dfrac{7\sqrt{2} - \sqrt{3}}{2\sqrt{2} - \sqrt{3}}$ in the form $m + \sqrt{n}$, where m and n are integers. *(4 marks)*

4 (a) (i) Express $x^2 - 6x + 11$ in the form $(x - p)^2 + q$. *(2 marks)*

(ii) **Use the result from part (a)(i)** to show that the equation $x^2 - 6x + 11 = 0$ has no real solutions. *(2 marks)*

(b) A curve has equation $y = x^2 - 6x + 11$.

(i) Find the coordinates of the vertex of the curve. *(2 marks)*

(ii) Sketch the curve, indicating the value of y where the curve crosses the y-axis. *(3 marks)*

(iii) Describe the geometrical transformation that maps the curve with equation $y = x^2 - 6x + 11$ onto the curve with equation $y = x^2$. *(3 marks)*

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5 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 - 4x^2 - 3x + 18$$

(a) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x + 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x - 3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Express $\mathrm{p}(x)$ as a product of linear factors. *(3 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 3x + 18$, stating the values of x where the curve meets the x-axis. *(3 marks)*

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6 The gradient, $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, of a curve at the point (x, y) is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 10x^4 - 6x^2 + 5$$

The curve passes through the point $P(1, 4)$.

(a) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(3 marks)*

(b) Find the equation of the curve. *(5 marks)*

7 A circle with centre $C(-3, 2)$ has equation

$$x^2 + y^2 + 6x - 4y = 12$$

(a) Find the y-coordinates of the points where the circle crosses the y-axis. *(3 marks)*

(b) Find the radius of the circle. *(3 marks)*

(c) The point $P(2, 5)$ lies outside the circle.

(i) Find the length of CP, giving your answer in the form $\sqrt{n}$, where n is an integer. *(2 marks)*

(ii) The point Q lies on the circle so that PQ is a tangent to the circle. Find the length of PQ. *(2 marks)*

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8 A curve has equation $y = 2x^2 - x - 1$ and a line has equation $y = k(2x - 3)$, where k is a constant.

(a) Show that the x-coordinate of any point of intersection of the curve and the line satisfies the equation

$$2x^2 - (2k + 1)x + 3k - 1 = 0$$

(1 mark)

(b) The curve and the line intersect at two distinct points.

(i) Show that $4k^2 - 20k + 9 > 0$. *(3 marks)*

(ii) Find the possible values of k. *(4 marks)*